

Contents

1	Markov semigroups in \mathbb{R}^N	1
1.1	The Cauchy problem and the semigroup	2
1.2	The weak generator of $T(t)$	9
1.3	Schrödinger operators via form method	11
1.3.1	From forms to semigroups	11
1.3.2	Contractivity properties	14
1.3.3	Symmetric forms	16
1.3.4	Ultracontractivity	17
2	Kernel estimates for Markov semigroups	21
2.1	Kernel estimates for a class of Kolmogorov semigroups	21
2.1.1	L^1 - estimates of some Lyapunov functions	23
2.1.2	Integrability and regularity results for the kernel	27
2.1.3	Pointwise estimates of kernels	34
2.2	Heat kernel bounds for Schrödinger operators	42
2.2.1	Integrability of Lyapunov functions	43
2.2.2	Regularity for parabolic problems and some interpolative estimates	45
2.2.3	Pointwise estimates on kernels	47
2.2.4	Small time estimates	51
2.2.5	Large time estimates	55
3	Kernel estimates for a class of Schrödinger semigroups	57
3.1	Introduction	57
3.2	Pointwise estimates of kernels	58
3.3	Estimates in space-time regions	62
3.4	The asymptotic distribution of the eigenvalues	64
4	Ultracontractivity of Schrödinger semigroups	71
4.1	Kernel estimates for a class of Kolmogorov operators	71
4.2	Intrinsic ultracontractivity for e^{-tH}	76

5	Parabolic Schrödinger operators	81
5.1	The parabolic reverse Hölder classes	82
5.2	Definition of the operator and some properties	84
5.3	Characterization of the domain of \mathcal{A}	88
5.3.1	The operator \mathcal{A} on L^1	88
5.3.2	A priori estimates in $L^p(\mathbb{R}^{N+1})$	89
A	Embedding Theorems and Solvability of Cauchy problems	95
B	The Karamata Theorem	99
C	An inequality in Sobolev spaces	103
D	A boundedness criterion	107
D.1	Shen's Theorem	107
D.2	An application of Shen's Theorem	118